Hierarchical Encoding in Visual Working Memory: Ensemble Statistics Bias Memory for Individual Items

Timothy F. Brady and George A. Alvarez

Abstract

Influential models of visual working memory treat each item to be stored as an independent unit and assume that there are no interactions between items. However, real-world displays have structure that provides higher-order constraints on the items to be remembered. Even in the case of a display of simple colored circles, observers can compute statistics, such as mean circle size, to obtain an overall summary of the display. We examined the influence of such an ensemble statistic on visual working memory. We report evidence that the remembered size of each individual item in a display is biased toward the mean size of the set of items in the same color and the mean size of all items in the display. This suggests that visual working memory is constructive, encoding displays at multiple levels of abstraction and integrating across these levels, rather than maintaining a veridical representation of each item independently.

Keywords:
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Observers can quickly and accurately compute ensemble statistics about a display, such as the mean size (Ariely, 2001; Chong & Treisman, 2003), orientation (Parkes, Lund, Angelucci, Solomon, & Morgan, 2001), and location (Alvarez & Oliva, 2008) of the items; the mean expression of a set of faces (Haberman & Whitney, 2007); and even higher-level spatial layout statistics (Alvarez & Oliva, 2009). However, little work has explored why observers compute these statistics and, in particular, whether the encoding of these higher-order statistics might play a role in how observers represent the individual items from such displays in memory.

Nearly all studies of visual working memory use displays consisting of simple stimuli and items that have been chosen randomly. These displays are, as best as possible, prevented from having any overarching structure or gist. Thus, influential models of visual working memory tend to treat each item as an independent unit and assume that items do not influence each others’ representations (Alvarez & Cavanagh, 2004; Bays, Catalao, & Husain, 2009; Luck & Vogel, 1997; Rouder et al., 2008; Wilken & Ma, 2004; Zhang & Luck, 2008; although see Lin & Luck, 2008, and Johnson, Spencer, Luck, & Schöner, 2009).

We propose that, contrary to the assumptions of previous models of visual working memory, ensemble statistics allow observers to encode such working memory displays more efficiently: In a process paralleling how people encode real scenes (Lampinen, Copeland, & Neuschatz, 2001; Oliva, 2005), observers might encode the “gist” of simple working memory displays (ensemble statistics such as mean size) in addition to information about specific items (their individual information). Such hierarchical encoding would allow observers to represent information about every item in the display simultaneously, significantly improving the fidelity of their memory representations compared with encoding only three or four individual items.

To test this hypothesis, we used the ensemble statistic of mean size and the grouping principle of common color, both of which are known to be automatically and effortlessly computed and could act as a form of higher-order structure in visual displays (Chong & Treisman, 2005b). Our results demonstrate a form of hierarchical encoding in visual working memory: The remembered size of individual items was biased toward the mean size of items of the same color and the mean size of all items in the display. This suggests that, contrary to...
existing models of visual working memory, items are not recalled as independent units; instead, an item’s reported size is constructed by combining information about that specific item with information about the set of items at multiple levels of abstraction.

**Experiment 1: Ensemble Statistics Bias Size Memory**

We examined whether the ensemble statistics of a display would bias memory for individual items when observers attempted to remember the size of multiple colored circles. We hypothesized that in the case of displays with both small red circles and large blue circles, observers would tend to report the size of a particular circle as larger if it was blue than if it was red. Such a size bias would suggest that observers had taken into account the size of the items in each color set.

**Method**

**Observers.** Twenty-one observers were recruited and run using Amazon Mechanical Turk (https://www.mturk.com). All were from the United States, gave informed consent, and were paid $0.40 for approximately 3 min of their time.

**Procedure.** All observers were presented with the same 30 displays consisting of three red, three blue, and three green circles of varying size (see Fig. 1) and were told to remember the size of all of the red and blue circles, but to ignore the green circles. We included the green distractor items in the displays because we believed they would encourage observers to encode the items by color, rather than to select all of the items into memory at once (Halberda, Sires, & Feigenson, 2006; Huang, Treisman, & Pashler, 2007). The order of the 30 displays was randomized across observers. Each display appeared for 1.5 s and was followed by a 1-s blank, after which a single randomly sized circle reappeared in black at the location that a red or blue circle had occupied. Observers had to slide the computer mouse up or down to resize this new black circle to the size of the red or blue circle they had previously seen at that location; they then clicked to lock in their answer and start the next trial.

**Stimuli.** The nine circles appeared on a gray background that measured 600 × 400 pixels. Each circle was positioned at a random location within an invisible 6 × 4 grid; jitter of ±10 pixels was added to the circles’ locations to prevent collinearities. The size and resolution of observers’ computer monitors were not controlled. However, all observers attested to the fact that the displays were visible in their entirety. Moreover, the critical comparisons are within subjects, and individual differences in absolute size of the displays are factored out by focusing on within-subjects comparisons between conditions.

Circle sizes were drawn from a separate normal distribution for each color. The mean diameter for the circles of a given color was chosen uniformly on each trial from the interval (15 pixels, 95 pixels), and the diameter of each individual circle was then chosen from a normal distribution with this mean and a standard deviation equal to one eighth of this mean. Thus, on a given trial, the three red circles could be sampled from around 35 pixels, the blue circles from around 80 pixels, and the green circles from around 20 pixels. However, which color set was largest and which was smallest was chosen randomly on each trial; thus, on the next trial, it could be the green circles that were largest and the blue circles that were smallest.

So that we could directly test the hypothesized bias in reported size, we generated 15 matched pairs of displays. First, 15 displays were generated as described; then, another 15 were created by switching the color of the to-be-tested item to the other nondistractor color (and making the reverse switch for another circle, so that there would still be three red circles and three blue circles in the display). Thus, the displays in
each of the pairs were matched in the size of all of the circles present and differed only in the color of two circles, including the circle that would later be tested. The 30 displays were randomly interleaved, with the constraint that paired displays could not appear one after the other. By comparing reported size when the tested item was one color with reported size when it was another color, we were able to directly test the hypothesis that observers’ memory for size is biased toward the mean size of all items in the tested item’s color set.

Results

Overall accuracy. We first assessed whether observers were able to accurately perform the size memory task by comparing their performance with an empirical measure of chance performance obtained by randomly pairing a given observer’s responses with the correct answers from different trials (mean difference by chance = 30.5 pixels, SEM = 0.78 pixels). Observers’ average error was 16.4 pixels (SEM = 1.7 pixels), a level of performance that was significantly better than our measure of chance, $p < 10^{-9}$.

Bias from same-colored circles. To test our main hypothesis, we examined whether observers’ size estimates tended to be biased toward the size of the circles with the same color as the tested circle. We divided each matched pair on the basis of which of the pair contained a tested item the same color as the circles that were smaller on average and which contained a tested item the same color as the circles that were larger on average. We then divided reported sizes on the latter trials by reported sizes on the former trials. A ratio of 1.0 would indicate that observers were not biased. However, if observers’ size estimates were biased toward the mean size of the circles in the same color as the tested item, this ratio would be greater than 1.0 (see Fig. 2a).

On average, the reported size of the tested circle was 1.11 times greater (SEM = 0.03) on trials with the larger same-colored circles than on trials with the smaller same-colored circles (see Fig. 2b). This ratio was significantly greater than 1.0, $t(20) = 4.17, p = .0004$. In addition, the direction of the effect was highly consistent across observers, with 19 of the 21 observers having a ratio above 1.0. The maximum possible bias was 1.6, because the larger same-colored circles were on average 1.6 times larger than their matched counterparts. Thus, the observers reported a size 18% of the way between the correct size and the mean size of the same-colored circles. This effect was a result of memory and not a perceptual bias, because in a version of the experiment with a precue indicating which item would be tested, observers ($N = 22$) reported the size accurately (mean error = 6.4 pixels, SEM = 0.5 pixels) and with no bias toward the mean size of the same-colored circles (bias = 1.00, SEM = 0.01; for details, see Perceptual Effects in the Supplemental Material available online).

Model: optimal integration across different levels of abstraction. One interpretation of the data is that observers represented the displays at multiple levels of abstraction and integrated across these levels when retrieving the size of the
tested circle or when initially encoding its size. To more directly test this idea, we formalized how observers might represent a display hierarchically using a probabilistic model (for similar models, see Hemmer & Steyvers, 2009; Huttenlocher, Hedges, & Vevea, 2000). The model had three levels of abstraction, representing particular circles, all circles of a given color, and all circles in the entire display. In the model, observers encoded a noisy sample of the size of each individual circle, and the size of each circle was itself considered a noisy sample from the expected size of the circles of that color, which was itself considered a sample of the expected size of the circles in a given display. We asked what observers ought to report as their best guess about the size of the tested circle (assuming normal distributions at each level).

The intuition this model represents is fairly straightforward: If the red circles in a particular display are all quite large, but the observer encodes a fairly small size for one of them, it is more likely that this circle is a large circle the observer accidentally encoded as too small than that it is a small circle the observer accidentally encoded as too large. Thus, in general, the model suggests that the optimal way to minimize errors in responses is to be biased slightly (either when encoding the circles or when retrieving their size) toward the mean of both the set of circles of the same color as the tested circle and the overall mean of the display. Figure 3 presents predictions of this model, along with a corresponding representation of the behavioral data from Experiment 1 (for information on model implementation, see Optimal Observer Model in the Supplemental Material available online). Note that in both the observers’ data and the model predictions, the slopes of the lines indicate a bias toward reporting all circles as less extreme in size than they really were, and also note that the plotted points indicate a bias toward reporting a size similar to the size of the same-colored circles.

The model had a single free parameter, which indicated how noisy the encoding of a given circle was (the standard deviation of the normal distribution from which the encoded size was sampled) and thus how biased toward the means observers’ size estimates ought to be. We set this parameter to 25 pixels (the estimated standard deviation of errors collapsing across all observers and displays), rather than maximizing the fit to the data. Although not strictly independent of the data being fit, this method of choosing the value for the parameter is not based on the measures we used to assess the model.

In general, the model provided a strong fit to the data as assessed by two different metrics: First, the model predicted the difference between the correct answer and reported answer for each display, ignoring the paired structure of the displays ($r = .89$, $p < .0001$). Second, the model predicted the difference in reported size between matched displays ($r = .82$, $p < .001$), even though the tested circle was actually the same size within each such pair. Any model of working memory that treats items
as independent (e.g., most slot and resource models, including the mathematical model presented by Zhang & Luck, 2008) cannot predict a systematic difference between these trials.

Discussion

We found that observers are biased by the ensemble statistics of a display when representing display items in visual working memory. In the case of displays with circles of several different colors, observers’ reports of the size of a given circle are biased by the size of the other circles of the same color. This effect is not accounted for by perceptual biases or location noise or by incorrectly reporting the wrong item from the display (see Potential Reports of the Incorrect Item in the Supplemental Material), is not a result of guessing on the basis of the mean size of the circles of that color (see Comparison to an Across-Trial Guessing Model in the Supplemental Material), and is compatible with a simple Bayesian model in which observers integrate information at multiple levels of abstraction to form a final hypothesis about the size of the tested item.

Experiments 2a and 2b: Attention to Color Is Required

In Experiment 1, the color of the items was task relevant. In fact, because observers have difficulty attending to more than a single color at a time (Huang et al., 2007), observers in Experiment 1 likely had to separately encode the sizes of the red circles and the sizes of the blue circles, and this might have increased the salience of color grouping. Salience of color as a grouping dimension may have been a crucial part of why observers used the mean size of the circles that were the same color as the tested circle in guiding their memory retrieval. Thus, in Experiments 2a and 2b, we removed the green circles from the displays and asked observers to simply remember the sizes of all of the circles. This allowed us to evaluate the automaticity of the biases we observed in Experiment 1—for example, the extent to which they depend on attentional selection and strategy. In addition, Experiments 2a and 2b provided a control for low-level factors that could have influenced the results of Experiment 1.

Method

Twenty-five new observers completed Experiment 2a, and 20 different observers completed Experiment 2b.

The methods and 30 displays used in Experiment 2a were the same as in Experiment 1 except that the green circles used as distractor items were not present in the displays. Experiment 2b was identical to Experiment 2a except that the circles were shown for only 350 ms rather than 1.5 s in order to decrease observers’ performance to the same level as in Experiment 1.

Results

Experiment 2a. Observers’ performance in Experiment 2a was very good. The average error was 10.2 pixels (SEM = 0.60 pixels), which was significantly less than our empirical measure of chance (29 pixels, SEM = 0.28 pixels) \(p < 10^{-19}\), and significantly less than the error of subjects in Experiment 1, \(t(44) = 3.73, p < .001\).

Observers in Experiment 2a displayed no bias as a function of color. The mean bias of 0.99 (SEM = 0.01) was not significantly different from 1.0, \(t(24) = -0.86, p = 0.39\) (see Fig. 2b). This result is compatible with the idea that the size of the same-colored circles does not bias observers’ estimates of the tested circle’s size when color is not task relevant.

However, the observers in Experiment 2a had significantly lower error rates than the observers in Experiment 1. Thus, it is possible that the observers in Experiment 2a did not display a bias because they were able to encode all of the circles accurately as individuals. To initially examine this possibility, we compared the least accurate 50\% of observers in Experiment 2a with the most accurate 50\% of observers in Experiment 1. The error rates reversed (Experiment 1: mean error = 10 pixels; Experiment 2a: mean error = 13 pixels), yet the bias remained present only in Experiment 1 (Experiment 1: 1.07; Experiment 2a: 1.00). This provided preliminary evidence that the difference in accuracy between the two experiments did not drive the difference in bias.

Experiment 2b: overall accuracy and bias. Experiment 2b experimentally addressed the concern that the lack of bias among observers in Experiment 2a was driven by their high performance level. In Experiment 2b, display time was reduced from 1.5 s to 350 ms to increase the error rate while maintaining the task irrelevance of color. Observers in Experiment 2b had an error rate of 15.9 pixels on average (SEM = 2.25 pixels). This was significantly less than our empirical measure of chance (31.3 pixels, SEM = 1.13 pixels), \(p < 10^{-9}\), but not significantly less than the error rate of subjects in Experiment 1, \(t(39) = -0.17, p = .86\). Thus, Experiment 1 and Experiment 2b were equated on error rate. However, observer’s reports of the size of the tested circles in Experiment 2b were still not biased toward the size of the same-colored circles (\(M = 1.00, SEM = 0.01\)).

Optimal integration model. We applied the same model used in Experiment 1 to the data from Experiment 2b, but with only two levels (no grouping by color): information about the particular circle and information about all the circles in the display. This model once again provided a strong fit to the experimental data (Fig. 4). Because the model did not use color information, it predicted exactly the same performance for both trials within each matched pair. This prediction is in line with
the observed bias of 1.00 in the experimental data. Furthermore, the model predicted the overall bias toward the mean size of the circles in the display, and its predictions correlated with the errors people made across all trials, \( r = .53, p = .002 \).

**Discussion**

In Experiments 2a and 2b, in which color was not task relevant, observers did not display a bias toward the mean size of the same-colored circles, even when the experiment was equated with Experiment 1 on difficulty. However, in both Experiment 2a and Experiment 2b, observers’ estimates were still biased toward the mean size of the circles in the display overall. The data are compatible with a Bayesian model in which observers treat all items as coming from a single group, rather than breaking the items into separate groups by color. Furthermore, the results of these experiments help rule out possible confounds in Experiment 1, such as the possibility that location noise caused swapping of items in memory, as the displays used in Experiments 2a and 2b were exactly the same as those used in Experiment 1 except for the absence of irrelevant green circles. We have also run Experiments 1 and 2a as separate conditions in a single within-subjects experiment and replicated the finding of a bias only when displays included the green circles (see Replication and a Within-Subject Experiment in the Supplemental Material).

**General Discussion**

We found that observers are biased by the ensemble statistics of a display when representing items in visual working memory. When asked to report the size of an individual circle, observers tended to report it as larger if the other items in the same color were large and smaller if the other items in the same color were small. This bias was reliable across observers and was predicted by a simple Bayesian model that encodes a display at multiple levels of abstraction. Taken together, our findings suggest that items in visual working memory are not represented independently and, more broadly, that visual working memory is susceptible to the very same hallmarks of constructive memory that are typical of long-term memory (Bartlett, 1932).

**Representation of ensemble statistics**

It is well established that the visual system can efficiently compute ensemble statistics (e.g., Alvarez & Oliva, 2009; Ariely, 2001; Chong & Treisman, 2003) and does so even when this is not required by the task, causing, for example, a false belief that an item with a set’s mean value of an attribute was present (de Fockert & Wolfenstein, 2009; Haberman & Whitney, 2009). However, less work has explored why the visual system represents ensemble statistics. One benefit of ensemble representations is that they can be highly accurate, even when the
local measurements constituting them are very noisy (Alvarez & Oliva, 2008, 2009). Another possible benefit of ensemble representations is that they can be used to identify outliers in a display (Rosenholtz & Alvarez, 2007), and thus potentially guide attention to items that cannot be incorporated in the summary for the rest of the group (Brady & Tenenbaum, 2010; Haberman & Whitney, 2009). The current work suggests a new use of ensemble statistics: Such statistics can increase the accuracy with which items are stored in visual working memory, reducing uncertainty about the size of individual items by optimally combining item-level information with ensemble statistics at multiple levels of abstraction.

It is interesting that observers in our experiments used the mean size of the circles of specific colors to reconstruct the displays only when color was task relevant, despite the fact that using this statistic would improve memory for the individual items in all conditions. This suggests that the units over which such ensemble statistics are computed is limited by selective attention (e.g., Chong & Treisman, 2005a). Turk-Browne, Jungé, and Scholl (2005) suggested that statistical learning, a form of learning about sequential dependencies, may happen automatically, but that the particular sets over which the statistics are computed may be controlled by selective attention. This hypothesis is compatible with our current findings: When observers did not attend to the colored sets as separate units, they may not have computed separate summary statistics for the two colored sets (alternatively, separate summary statistics may have been encoded, but not used in reconstructing the circle sizes). However, when observers attended to color, the ensemble statistics for the two colors seem to have been computed in parallel, as found by Chong and Treisman (2005b).

**Dependence between items in visual working memory**

Our results demonstrate a case of nonindependence between items in visual working memory: We found that items are represented not just individually, but also as a group or ensemble. Although previous experiments did not directly address such hierarchical effects, nonindependence between items in visual working memory has been observed previously. For example, Huang and Sekuler (2010) found that observers exhibit bias in reporting the spatial frequency of Gabor patches, tending to report spatial frequencies as though they have been pulled toward the spatial frequencies of previously presented Gabor patches. In addition, Jiang, Olson, and Chun (2000) have shown that changing the spatial context of an item influences memory for that item (see also Vidal, Gauchou, Tallon-Baudry, & O’Regan, 2005). This suggests that an item is not represented independently of its spatial context in working memory.

Similarly, several studies (Brady & Tenenbaum, 2010; Sanocki, Sellers, Mittelstadt, & Sulman, 2010; Victor & Conte, 2004) have shown that observers can take advantage of perceptual regularities in working memory displays to remember more individual items from those displays. Brady and Tenenbaum (2010) investigated checkerboard-like displays and conceptualized their findings in terms of hierarchical encoding, in which the gist of the display is encoded in addition to specific information about a small number of items that are least consistent with the gist. This hypothesis is compatible with the model we presented here for simpler displays, according to which observers encode ensemble information as well as information about specific items.

This dependence between items in memory is not predicted or explained by influential models of visual working memory. Current theories model visual working memory as a flexible resource that is quantized into slots (Zhang & Luck, 2008) or continuously divisible (Alvarez & Cavanagh, 2004; Bays & Husain, 2008; Wilken & Ma, 2004). According to these models, fewer items can be remembered with higher precision because they receive more memory resources. However, these models assume that items are stored independently, and therefore cannot account for the dependence between items in memory observed in the current study. Expanding these models to account for the current results will require specification of whether abstract levels of representation compete for the same resources as item-level representations (e.g., Feigenson, 2008), or whether there are essentially separate resources for ensemble representations and item-level representations (e.g., Brady & Tenenbaum, 2010).

**Role of long-term memory in visual working memory**

In addition to work demonstrating dependencies between stored representations of items and hierarchical encoding of a particular display, there is a significant amount of previous work showing that the representation of items in visual working memory depends on information in long-term memory (e.g., Brady, Konkle, & Alvarez, 2009). For instance, Konkle and Oliva (2007) and Hemmer and Steyvers (2009) have shown that knowledge of the size of an object in the real world biases the remembered size of that object in a display after a short delay. Hemmer and Steyvers (2009) provided a model of this effect as due to Bayesian inference in a constructive memory framework, and their model is similar to the one we proposed here for the on-line representation of the displays in our experiments. Convergence between a model for using ensemble information from the current display and a model for integrating information from the current display with information from long-term memory suggests a promising future direction for understanding the use of higher-order information in memory.

**Conclusion**

We found that observers are biased by the ensemble statistics of a display when representing items from that display in visual working memory. Rather than storing items independently, observers seem to construct the size of an individual item using information from multiple levels of abstraction. Thus, despite
the active maintenance processes involved in visual working memory, it appears to be susceptible to the very same hallmarks of constructive memory that are typical of retrieval from long-term memory and scene recognition (Bartlett, 1932; Lampinen et al., 2001). Cognitive and neural models of visual working memory need to be expanded to account for such constructive, hierarchical encoding processes.

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Supplemental Material
Additional supporting information may be found at http://pss.sagepub.com/content/by/supplemental-data

Note
1. See Hollingworth (2008) for studies that employed real-world scenes.

References


SUPPORTING MATERIAL

Replication and a Within-Subject Experiment

Running the experiment on the internet allowed for variation in the visual angle of the dots and meant that each observer saw only 30 trials\(^1\). Thus, we ran a control experiment in the lab with 6 observers using the same paradigm. Observers saw 400 trials each (200 matched pairs). These observers in the lab showed the same effects as observers tested on Mechanical Turk. They had a mean error of 20.2 pixels and a bias of 1.04, significantly greater than 1.0 (\(t(5)=3.47, p=0.02\)). The maximum possible bias was 1.37, since the same-colored dots were on average 1.37 times larger in the larger of the matched trials than the smaller. Thus the observers run in the lab reported a size 11% of the way between the correct size and the mean of the same colored dots.

In addition to replicating the experiments in the lab, we also replicated our main results on Mechanical Turk. In particular, to bolster the evidence for our effect we have run both a within-subject experiment (\(N=17\)) and replicated both the between-subject experiments (\(N=16\) and \(N=26\), respectively; all conducted on Mechanical Turk). In the within-subject experiment, we combined Exp. 1 with Exp. 2A within observers (thus observers performed 60 trials, 30 with green dots and 30 without green dots present). We found a bias of 1.11 (SEM 0.02) in the trials with green dots and a bias of 1.02 (SEM

\(^1\) In general we find observers on Mechanical Turk to be slightly better at our tasks, on average, than observers in the lab. For example, in a replication of Luck & Vogel (1997) we find capacities of 4.2 colors on average in 65 online observers.
0.02) for trials without green dots, a significantly larger bias on green dot trials within-subjects ($t(16)=2.90, p=0.01$). In addition, the bias was significant in the green-dot displays ($t(16)=4.40, p=0.0004$) but not the displays without green dots ($t(16)=0.82, p=0.42$).

In the between-subject replication of Experiment 1 with a different set of displays and different observers, the average bias was 1.09 ($N=16$), with SEM 0.03. The difference from no bias (1.0) was significant: $t(15)=2.29; p=0.037$. In the replication of Experiment 2B with a different set of displays and observers, the average bias was 1.00 ($N=26; \text{SEM} 0.016$), not significantly different than 1.00.

**Perceptual Effects**

Is the bias from same-colored items a result of memory or a perceptual effect caused by crowding or grouping principles in our display? To determine this, we ran a study that was identical to Experiment 1 except that 500ms before the onset of the dots, a single black 'X' appeared at the location of the dot that would later be tested. We instructed observers that this cue indicated which item would be tested (it was 100% valid). If observers have to encode only a single item from the display and know in advance which item will be tested, this should eliminate any bias resulting from memory processes. However, if the locus of our effect is perceptual observers should still be biased toward the size of the same-colored dots. Observers ($N=22$) reported the size accurately (error 6.4px, SEM 0.5px) and with no bias toward the mean size of the same-colored circles (bias: 1.00, S.EM. 0.01). This suggests the bias was a result of memory processes, not a perceptual effect from our display.
Potential Reports of the Incorrect Item

Using a similar paradigm but with continuous report of color rather than size, Bays, Catalao and Husain (2009) report that observers sometimes accidentally report the color of the wrong item, perhaps because of noise in their representation of the items' locations. Such location noise would not, in general, affect our conclusion that there is a bias toward the mean of the same colored dots. In particular, if swapping was simply a result of location noise, then since our matched displays contain the exact same size dots in the exact same locations, no difference could arise between them. However, it is possible that observers would be more likely to swap with items in the same color as the target item, and that this could account for the bias we find. If this were the case, we might expect a mixture of correct reports and reports of the incorrect items in our data, resulting in a multimodal distribution. To address this concern, we examined whether the location of the same-colored dots affected the bias we observed, and, additionally, used a mixture model similar to that reported by Bays, Catalao and Husain (2009) to directly examine the possibility of swapping with same-colored items.

To examine the effect of the location of the same-colored dots, we divided the matched pairs by the mean distance of the same-colored dots to the tested dot's location. On those display pairs in which the same-colored dots were much closer in location for one of the matched displays than the other, we might expect a larger bias. Instead, the correlation between the size of the bias and how differently located they were in the two display pairs was not significant, and in fact trended negative ($r=-0.27$, $p=0.33$) the opposite of the direction predicted from a swapping account.
As a second measure of the potential of swapping, this time ignoring the location of the items, we used a mixture model to estimate the percentage of swaps directly from the data, effectively examining its bimodality (Bays, Catalao & Husain, 2009). The mixture model attempted to parse the observers' responses into those most likely to have been noisy reports of the correct item, those most likely to have been random guesses, and those most likely to have been swaps2. Excluding all responses except those the model considered twice as likely to be noisy reports of the correct item than swaps or guesses still resulted in a substantial bias toward the mean size of the same colored items (M=1.05, SEM:0.016, difference from 1.0: $t$(20)=3.20, $p$=0.004). Note that this is an extremely conservative measure, since it effectively counts only responses that are closer to the size of the tested dot than the size of any other dot. Taken together, we believe these analyses help rule out explanations of our data in terms of location noise and reporting the size of the wrong item.

**Comparison to an Across-Trial Guessing Model**

Rather than performing an integration across different levels of representation on each trial, as proposed in our Bayesian integration model, it is possible that our results could arise from a model in which on some trials observers remember the dot and on other trials the observers' guess based on the dots color. For example, on trials in which

2 See Bays et al. (2009) for details of this modeling. We made two changes to their model. First, we separately modeled swaps with item in the same color and swaps with items in a different color (by adding another $\beta$). Additionally, since size, unlike color, is not circular, guessing cannot be treated as a uniform distribution. Instead, we use the empirical distribution of all responses of all observers across all trials as our guessing distribution.
the participant retains information about the size of the probed dot, it might be reproduced without bias. On other trials, in which the participant retains no size information about the probed dot, the participant might tend to guess something around the mean of the size of the dots the same color as the probed dot. We will refer to this model as the across-trial guessing model.³

While such a model requires observers encode the display at multiple levels of abstraction and integrate across these levels by choosing which kind of information to use in generating a particular response, it is significantly different than the within-trial Bayesian integration model we propose. We believe the evidence from the current experiments heavily supports the within-trial integration model.

First, the across-trial guessing model requires there to be a large number of trials where observers know the color of the tested dot but have no information at all about this dot's size. Both the original work of Luck & Vogel (1997) and important work by Brockmole and colleagues (Logie et al. 2009; Gajewski & Brockmole, 2006) demonstrates that not only is there a benefit to encoding all of the features of a single object, but that observers do so on nearly all trials and represent the objects as bound units. A model which requires observers to frequently know only a single feature of an object is thus theoretically unlikely and in conflict with existing data on binding in visual working memory.

Second, as reported above in the section on modeling location noise and potential item swaps, we can examine trials which are unlikely to have been guess trials by looking at only responses that are closer in size to the size of the correct dot than to the size of

³ We thank an anonymous reviewer for this suggestion.
any of the other dots (including those of the same color). This still results in a substantial bias toward the mean size of the same-colored dots (see results in location noise section). This is contrary to what you would expect from the across-trial guessing model, which posits a bias arising only from trials where observers do not know the size of the tested dot.

Finally, using model comparison techniques, we can directly compare the distributions predicted by the two models. The within-trial Bayesian integration model assumes the distribution of sizes observers' report for a particular dot has a peak that is shifted toward the mean size of dots of the same color, whereas the across-trial guessing model proposes a mixture between correct responses and responses that are drawn from a distribution around the mean size of the same-colored dots.

The Bayesian model has only a single parameter, the standard deviation of observers' encoding error (this parameter decides both how noisy the distribution is and how much the specific item information is integrated with the ensemble size information). The across-trial guessing model has two parameters, the standard deviation of observers' encoding error and the percentage of trials in which observers report from a distribution around the same-colored mean rather than the correct dot (the guessing rate). In addition, we can choose to make the guessing distribution a normal distribution with the true standard deviation of the dots within the same color, or increase the variance based on the expected sampling error.

For each subject, we performed a leave-one-trial-out cross-validation to find the maximum likelihood parameters for each model. Then we computed the log-likelihood of the observer's response on the left out trial using those parameters. Averaging across all
possible left out trials gives us the log-likelihood of each of the two models for each observer. Finally, we can compare these log-likelihoods using AIC (Akaike Information Criterion; Akaike, 1974)\(^4\). This gives us an AIC score for each model for each observer (lower AIC values indicate a better model fit). We find that across observers, the AIC for the Bayesian model consistently indicates a better fit than the AIC for the across-trial guessing model. This is true both if we assume the guessing distribution is simply a normal with the mean and standard deviation of the true size of the dots of the same color (Bayesian model AIC = 10.8, SEM 0.2, Discrete-guessing model AIC = 13.8, SEM 0.8, \(t(20)=-3.95, p<0.001\)) or if we increase this standard deviation by adding in the variance from sampling each dot's size (Discrete-guessing model AIC = 12.7, SEM 0.18, \(t(20)=-26.8, p<10^{-16}\)). In fact, using AIC the within-trial Bayesian integration model is preferred in every single observer. Moreover, it is preferred on average even if we do not use AIC to adjust for the greater flexibility of the across-trial guessing model (the log-likelihood of the within-trial integration model is significantly higher than the version of the across-trial guessing model adjusted for measurement error, \(t(20)=2.23, p=0.038\)). Thus, in spite of the greater flexibility of the across-trial guessing model, it does not fit the data as well as the within-trial Bayesian integration model.

\(^4\) We choose to compare models using AIC because it penalizes more complex models (e.g., models with more parameters, like the across-trial guessing model in this case) less than other model comparison metrics.
Optimal Observer Model

To more directly test the idea that observers represent the display at multiple levels of abstraction and integrate across these levels when retrieving the size of the tested dot, we formalized this theory in a probabilistic model. In the model, observers are assumed to get a single noisy sample from each of the 9 dots on the screen (sampled from a normal distribution centered around the size of the dot and with a standard deviation of 25px). Then, the observer attempts to infer the size of each of the dots on the screen using these samples. A naïve, non-hierarchical model simply treats each of the dots independently and thus report the size of each dot as the size that was sampled for that dot. As an alternative, we present a hierarchical Bayesian model that pools information from all of the dots to best estimate the size of any given individual dot. It does so by representing the display at two additional levels of abstraction and partially pooling information at each of these levels: (1) all dots of the same color; (2) all dots on the display. By assuming that dots of the same color and all the dots on a display are sampled from some underlying distribution and therefore provide mutual information about each other, such a model arrives at a more accurate estimate of the size of each dot. Such models are standard in Bayesian statistics (Gelman, Carlin, Stern & Rubin, 2003) and have been previously applied to similar problems in cognitive science (Huttenlocher et al. 2000; Hemmer & Steyvers, 2009).

Formally, we assume that observers' treat the dots of a given color as sampled from a normal distribution with unknown mean and unknown variance, and additionally

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5 If you assume they sample only a random subset of 3-4 dots the model predictions remain qualitatively the same.
treat these distributions' means as coming from an overall normal distribution that pools information across all of the colors. We put uniform priors over the reasonable range of possible sizes (0-200 pixels) on the parameters of these normal distributions. The exact model is represented in WINBUGS as follows. Note that the normal distribution in WINBUGS is parameterized by a mean and a precision, rather than a mean and standard deviation; nevertheless we put a uniform prior on standard deviation, which is a more standard model (Gelman, Carlin, Stern & Rubin, 2003).
WinBUGS code for the model in Experiment 1:

model
% C = number of colors,
% L = number of dots of each color.
% We observe 'sample'.
{ 
  overallMean ~ dunif(0,100)
  overallMeanStd ~ dunif(0,100)
  overallMeanPrec <- 1/(overallMeanStd*overallMeanStd)

  overallStd ~ dunif(0,100)
  overallStdStd ~ dunif(0,100)
  overallStdPrec <- 1/(overallStdStd*overallStdStd)

  stdev <- 25
  precision <- 1/(stdev*stdev)

  for (i in 1:C)
  { 
    groupMean[i] ~ dnorm(overallMean, overallMeanPrec)
    groupStd[i] ~ dnorm(overallStd, overallStdPrec)
    groupPrec[i] <- 1/(groupStd[i]*groupStd[i])
  }

  for (i in 1:C)
  { 
    for (j in 1:L)
    { 
      dotMean[i,j] ~ dnorm(groupMean[i], groupPrec[i])
      sample[i,j] ~ dnorm(dotMean[i,j], precision)
    }
  }
}

WinBUGS code for the model in Experiment 2:

model
{ 
  overallMean ~ dunif(0,200)
  overallMeanStd ~ dunif(0,100)
  overallMeanPrec <- 1/(overallMeanStd*overallMeanStd)

  stdev <- 10
  precision <- 1/(stdev*stdev)

  for (i in 1:L)
  { 
    dotMean[i] ~ dnorm(overallMean, overallMeanPrec)
    sample[i] ~ dnorm(dotMean[i], precision)
  }
}
References


